Noise-Shaped Predictive Coding for Multiple Descriptions of a Colored Gaussian Source

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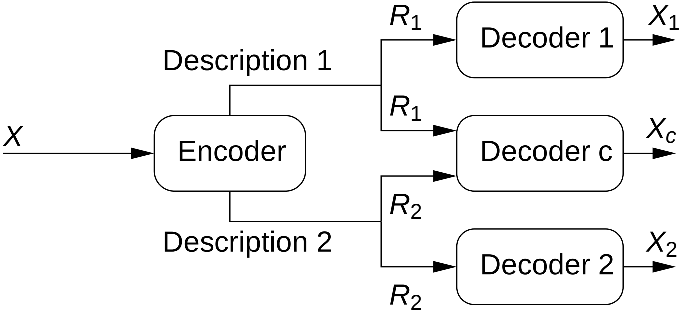
Outline

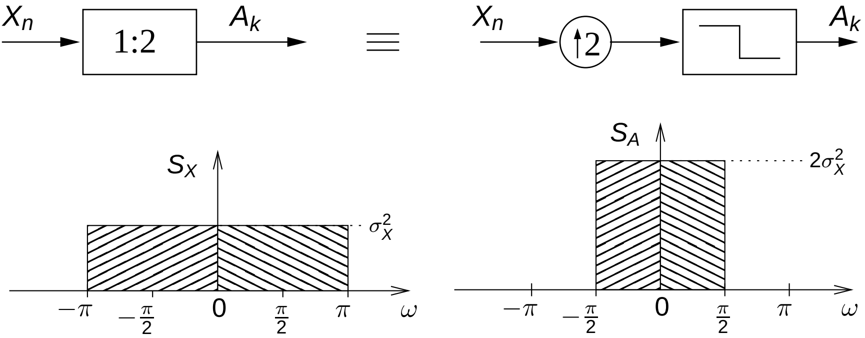
Setup



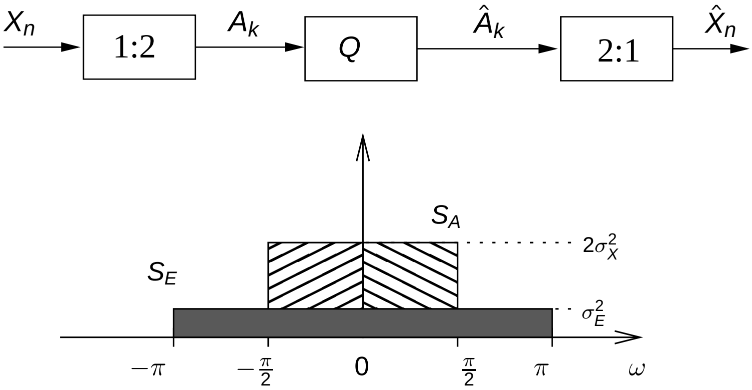
- Two descriptions
- Symmetric case: $R_1 = R_2$ and $D_1 = D_2$
- Discrete-time white Gaussian scalar process (i.i.d.)

Oversampling by two

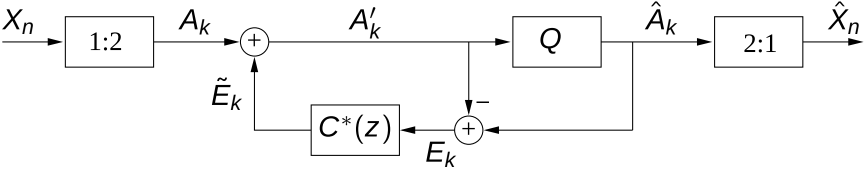
- Ideal upsampling can be done by inserting a zero between every sample, and then apply an ideal lowpass filter (sinc function)
- We use n for indexing at original rate and k for the upsampled rate



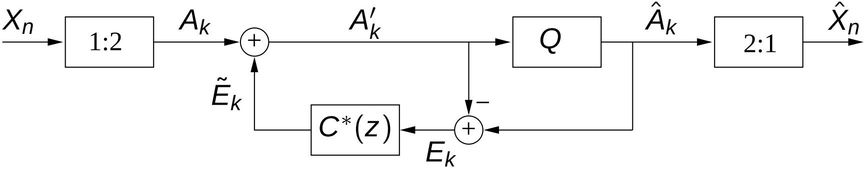
Oversampling and Dithered Quantization



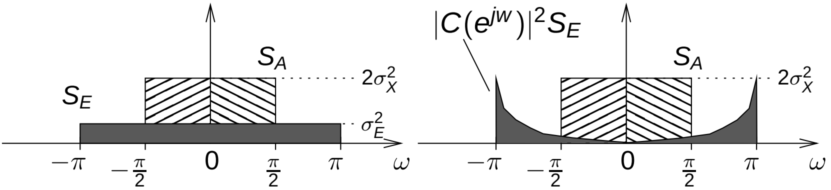
- Q is an ECDQ $\Rightarrow$ flat power spectrum through the complete frequence band



- $C^*(z) = \sum_{i=1}^{p} c_i z^{-i}$
- $C(z) = 1 + C^*(z)$ is the noise shaping filter of order p



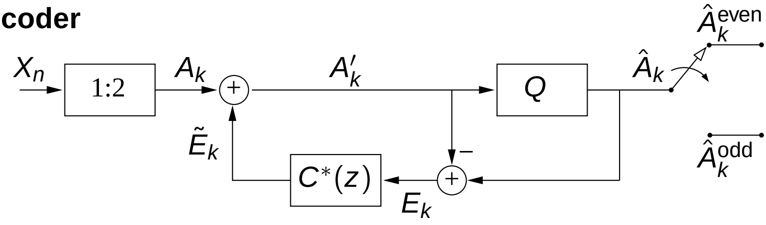
- $C^*(z) = \sum_{i=1}^{p} c_i z^{-i}$
- $C(z) = 1 + C^*(z)$ is the noise shaping filter of order p
- Noise is shaped away from the *in-band* spectrum



- This is standard Delta-Sigma Quantization

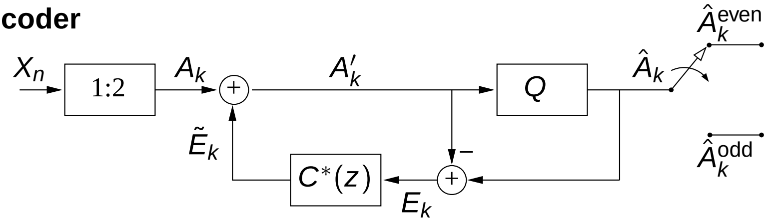
Multiple Description by ΔΣ Quantization

- **Encoder**

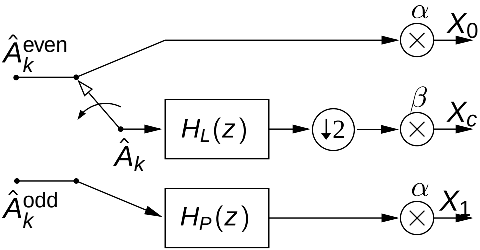


Multiple Description by ΔΣ Quantization

- **Encoder**

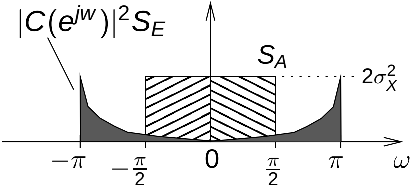


- **Decoder**



Central and Distortions of ΔΣ MDC

- Central distortion is given only by the *in-band* quantization noise



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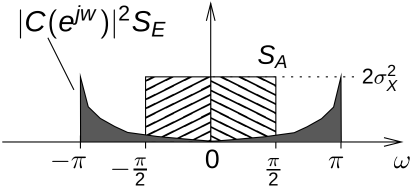
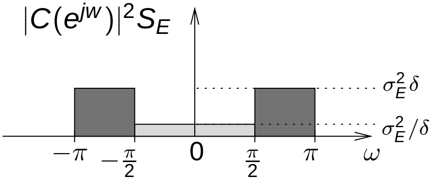


- Creation of side descriptions ⇒ downsampling by 2
- But no **anti-aliasing filter**!
- Thus, side distortion is given by the *in-band* and the *out-of-band* noise spectrum

Central and Distortions of ΔΣ MDC

- Filter order $p \to \infty \Rightarrow$ brick-wall filter is possible
- This choice guarantees that $C(z)$ is minimum phase



- $D_c = \frac{1}{2}\sigma_E^2/\delta$ (at high resolutions)
- $D_0 = D_1 = \frac{1}{2}\sigma_E^2(\delta + 1/\delta)$
- By adjusting $\delta > 1$ we trade-off side for central distortion
- $R \approx \frac{1}{2}\log_2(\sigma_X^2/\sigma_E^2)$ per even/odd sample

Key Results

Theorem:

- Asymptotically, as $p \to \infty$ and VQ dimension $\to \infty$

  and or any coding rate:

1) The entropy rate and the distortion levels of the ΔΣ Quantization scheme achieve the symmetric two-channel MD rate-distortion function for a memoryless Gaussian source and MSE metric
2) the optimum noise-shaping filter is unique, minimum phase, and its magnitude spectrum is piecewise flat with a single jump discontinuity at $\omega = \pi/2$

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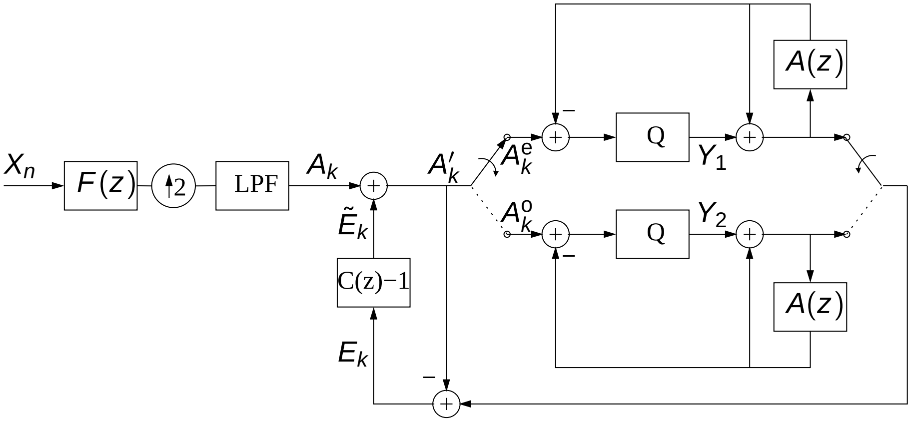
Lemma:

- We have obtained a closed-form expression for the unique optimum p^{th}-order noise-shaping filter

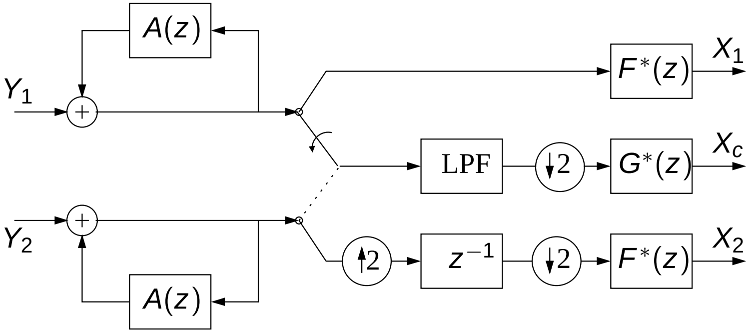
MD ΔΣ Quantization for Colored Gaussians

- It is known that the RDF of a colored Gaussian source can be achieved be *prediction* [Zamir et al.'07]
- The Multiple-Description RDF for a white Gaussian source, can be achieved by oversampling and *noise-shaping*
- We will combine Differential Pulse Coded Modulation (DPCM) with Delta Sigma Quantization
- Specifically, we embed two DPCM loops, within a common noise-shaping loop

DSQ-DPCM Encoder



DSQ-DPCM Decoder



Key results

Theorem:

- The proposed architecture, can achieve the MD-RDF for arbitrarily colored Gaussian sources for any given admissible side and central distortion levels

Important observations:

- The quantizer is a high-dimensional VQ, so that the quantization noise is Gaussian in the conventional divergence sense
- The side post-filters are *conjugates* of the pre-filter, hence, they are **not** *causal*

Conclusions and Future Research Directions

- We showed that the quadratic Gaussian MD-RDF can be achieved by *noise-shaping* combined with *prediction*
- The *noise-shaping* filter controls the distortion trade-off between the central and side descriptions
- The *prediction* loops, take care of the source memory in the side descriptions

- Would be interesting to *"causalify"* the proposed solution, to avoid acausal post-filters
- Multiple-descriptions in closed-loop control systems?